Solar Radiometry of the Microwave Emitting Region of the Sun

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Abstract

The temperature of the Sun in degrees Kelvin was measured via solar radiometry and mathematical techniques regarding the resolving power of a telescope. A microwave radiometer was calibrated using a known logarithmic relationship between measured signal of microwave noise and power, which itself we assert is proportional to temperature. Next, the resolving power of the detector unit of the microwave radiometer was calculated both experimentally using a geosynchronous satellite as well as based on the Rayleigh Criterion and the dimensions of the dish. When using the experimentally derived value of the resolving power of the dish, 6.65±0.11°, the temperature of the microwave emitting region of the sun was determined to be $331,000 \pm 68,000$K. Calculation using the value determined from the Rayleigh Criterion, 3.68±0.11°, yielded a value of $101,500 \pm 20,800$K .

Introduction and Relevant Theory

Radiometry is the use of waves and wave physics to study the temperature of objects at a distance. We note that the aperture size of a telescope determines its angular resolution for observing distant objects, like the Sun. This can be assumed to be a solid angle of radiation acceptance for a radiometer, $\Omega_{Antenna}$. To calculate $\Omega_{Antenna}$ for an optical device, one can rely on the Rayleigh Criterion proven by Young and Freedman[1], which is given by:

$$\Delta\theta = 1.2\frac{\lambda}{D} \tag{1}$$

where $\Delta\theta$ is the resolving power, λ is the wavelength of the resolved radiation, and D is the diameter of the circular aperture of the device. One can also experimentally measure $\Delta\theta$ by performing a half-power half-width normalization of the detected signal when observing a geosynchronous satellite. This will be elaborated on in greater detail in the following section.

We then approximate the three-dimensional solid angle $\Omega_{Antenna}$ by approximating that the dish's resolving geometry is circular rather than elliptical. Performing the necessary integral, we note that[2]:

$$\Omega_{Antenna} = \frac{\Delta\theta^2}{4\pi} \tag{2}$$

Kraus asserts that in the long wavelength limit of the blackbody radiation of a source the Rayleigh-Jeans law should hold- $T \propto P$[3], where T represents the temperature of the source and P represents power of the source (which itself responds logarithmically to the detector unit in the radiometer).

We also must account for the fact that the solid angle of the Sun is a fraction of $\Omega_{antenna}$, and thus we can see that the detected temperature of the source will be a fraction of its actual temperature. Exploiting the geometry of the dish, we have the relation that:

$$T_{Sun} = \frac{T \cdot \Omega_{Sun} + (T_{Sky} \cdot (\Omega_{Antenna} - \Omega_{Sun}))}{\Omega_{Antenna}} \tag{3}$$

where T is the outputted temperature after calibration of the radiometer, T_{Sky} is the background temperature of the sky given by the Cosmic Microwave Background radiation (3K), and Ω_{Sun} is the solid angle of the Sun as seen from the radiometer.

Experimental Setup and Results

Calibrating the LNB

The central apparatus of the experiment was the radiometer, which itself was made up of two distinct units: the microwave dish and the Low Noise Block (LNB) detector unit. The microwave dish is best approximated as a circular dish with diameter 46.5cm. The detector unit of the dish is fed via coaxial cable into the LNB, which transforms the signal into the $0.95 - 2.05$GHz range by beating the detected signal (which we assert to be in the $12.2 - 12.7$GHz range based on the wavelength of microwaves) against an 11GHz oscillator.

The signal outputted by the LNB responds logarithmically to the power of the source, and thus the LNB was calibrated using attenuators. The attenuators dampen the signal strength of the LNB and can be added in series to produce a fit such that the relationship between signal strength and power of the source is given by:

$$D = a + b \log_{10} P \tag{4}$$

where, recalling from above that $P \propto T$, D is the detected signal, and a and b are calibration constants intrinsic to the detector. The following table gives the attenuation of the LNB and the responding signal strength for several attenuations, constructed by placing attenuators in series (from the LNB manual, the uncertainty in the detected signal is uniformly 0.5):

Table 1: LNB Detector Signal at Attenuations

attenuation (dB)	detected signal
0	56.9
3	44.6
6	32.4
9	16.3
10	12.1
12	3.4
13	3.0

Plotting this relationship, we observe a linear relationship and perform a least-squares linear regression to derive the constants a to be 57.8 ± 0.11 and b to be -4.52 ± 0.85:

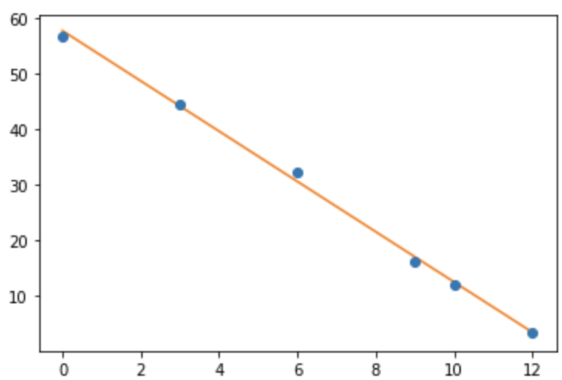

Figure 1: A plot of the measured signal vs. attenuation, with a linear regression best-fit line

Something we have not yet taken into account is the intrinsic noise of the detector. The LNB possesses an intrinsic noise that we can denote as the "noise temperature" T_0. So, in actuality, $P \propto T + T_0$. Extending equation 4, the complete relationship between detected signal from the radiometer and the temperature of the source is given by:

$$D = a + b \log_{10} \frac{T + T_0}{T_0 + T_1} \tag{5}$$

where a, b, and T_0 are the aforementioned constants intrinsic to the LNB and T_1 is room temperature (in this experiment 298.15K). In order to determine the noise temperature T_0, a known temperature of a source T was required to be used to relate the measured signal D to T_0. In this experiment, that known temperature was Eccosorb-lined styrofoam chamber filled with liquid nitrogen, which has a known temperature T of 77K. The detector was used on this insulated source and a signal strength was measured as 41.4 ± 0.5. Thus, rearranging equation 5 for T_0:

$$T_0 = \frac{T_1 10^\alpha - T}{1 - 10^\alpha} \tag{6}$$

where α is given by $\frac{D-a}{b}$ and T in this instance is specifically 77K, the known temperature of liquid nitrogen. Evaluating this we find that the intrinsic noise temperature of the LNB T_0 is 92.7 ± 17.4K.

Finding the Resolving Power of the Dish

In performing the above, the LNB portion of the radiometer was calibrated, but the resolving power of the dish must also be calculated using its dimensions to account for the fact that the Sun only takes up a fraction of the dish's acceptance, as described in equation 3. As mentioned in the previous section, the resolving power of the dish was calculated experimentally and based on the Rayleigh Criterion. The Rayleigh Criterion, as described in equation 1, with the dish's diameter D of 46.5cm and using the wavelength λ of microwaves 2.46cm, we get that the resolving power of the dish is 0.064 ± 0.002 rad, or 3.68 ± 0.11 deg.

The resolving power of the dish was also calculated experimentally. There is a geosynchronous satellite in the sky above McGraw Tower on Cornell University's campus in Ithaca, NY, where this experiment was conducted. This satellite, too, can act as a source of microwaves. By positioning the radiometer outside the Physical Sciences Building, and rotating the dish a known angle θ and measuring the detected signal, and then plotting this relationship we can normalize our resolving power using a half-width half-power normalization.

From the LNB manual, the uncertainty in the detector readings is uniformly 0.5 [4]. Plotting this data, we can perform a half-width, half-power normalization to obtain a measure of the resolving power of the dish experimentally (see figure 2 below). In doing so we find that the resolving power of the dish, $\Delta\theta$ is 6.65 ± 0.11 deg, or 0.12 ± 0.001 rad. Recalling from equation 2, we transform these two resolving angles into an experimental and theoretical value for $\Omega_{Antenna}$.

After all of these steps, the radiometer is fully calibrated. The radiometer was thus aimed at the Sun from the same point outside the Physical Sciences

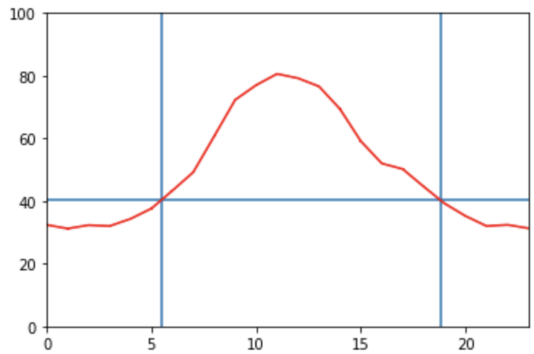

Figure 2: Plot of detected signal vs. angular offset of a geosynchronous sattelite. The horizontal line represents half-power and the distance between the vertical lines represent the full-width.

Building as well as Bailey Plaza for three separate trials each, resulting in a detected measurement of the Sun as 89.7 ± 1.2. Plugging this value into equation 5 and rearranging for T, using the resolving power calculated from the Rayleigh Criterion yields a value of $101,500 \pm 20,800$K. When using the experimental value, the temperature of the sun is calculated to be $331,000 \pm 68,000$K.

Analyzed Data and Conclusions

The resulting values for the temperature of the Sun are quite far from the accepted value. The accepted value of the Sun is 5000K[5]. If one were to propagate the measurements of the Sun as a normal distribution with a mean of the experimental value and standard deviation given by its uncertainty, then the accepted value of the Sun is 4.6 standard deviations away from the accepted value when using the Rayleigh Criterion's resolving power and is 4.8 standard deviations away from the accepted value when using the experimentally calculated resolving power. Even though the experimental value is three times farther away from the accepted value, the Rayleigh Criterion has three times smaller uncertainty, resulting in the similar standard deviations. If the experiment were to be repeated, this means that on repeated measurement using the radiometer there is a $\leq 0.1\%$ chance of encountering the accepted value of 5000K.

The accepted value of the temperature of the Sun is a very well-documented and well-studied measurement. It is unreasonable to assert that this experiment could usurp the known value of the temperature of the Sun, and by orders of magnitude at that, but it is reasonable to question whether the relevant theory and relationships were applicable in the lab.

The largest sources of statistical uncertainty in the experiment were the uncertainties in the fit parameters a and b of the calibrated LNB. Given that the LNB detected signal has a uniform uncertainty of ± 0.5, there is little statistical uncertainty in the detector readings, and the main source of uncertainty is the fit parameters.

If the experiment were to be repeated, perhaps the following assumptions/relations could be re-examined to reduce systematic uncertainty, although the degree to which is unclear. First, the temperature of the eccosorb lined styrofoam chamber was assumed to be 77K without any measurement with a thermometer due to a lack of equipment. This temperature could be validated to get a more accurate figure for the intrinsic noise temperature of the LNB detector unit. Furthermore, the dish was assumed to project a circular solid angle where it is in fact elliptical, so the calculation for equation 2 is not completely accurate. Finally, the fluorescent lights in the lab may have minimally contributed to the detector reading when calibrating.

Appendix

The fit parameters were calculated using a least squares linear regression, using the numpy python package.

The probability of achieving the accepted value was performed by assuming a Gaussian distribution of measurements with a mean of the experimental value and and a standard deviation of the uncertainty, and calculating how many standard deviations away from the mean the resulting measurement was according to:

$$\frac{|x - \mu|}{\sigma} \tag{7}$$

where x is the expected value and μ is the experimental value.

The uncertainty in the detected signal from the Sun was taken to be the square root of the sample variance of the data rather than the detected value of 0.5, as given by the formula

$$S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} \tag{8}$$

References

[1] Circular Apertures and Resolving Power, Young University Physics, accessed via Cornell course Canvas Website

[2] Proof of Solid Angle Transformation, Carl Franck, Cornell Physics 3310 Canvas Website, accessed October 26, 2021

[3] Introductory notes, Solar Radiometry, Cornell Physics 3310 Canvas website, accessed October 26, 2021.

[4] LNB Detector Manual, accessed via Cornell Physics 3310 Canvas Website, accessed October 26, 2021

[5] The Sun's Layers and Temperatures, NASA Website, accessed October 26, 2021. https://www.jpl.nasa.gov/nmp/st5/SCIENCE/sun.html