Attenuation Coefficients for Lead, Aluminum, and Copper at 700KeV

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Abstract

The attenuation coefficients for Lead, Aluminum, and Copper at 700KeV were obtained by measuring the amount of 700KeV gamma rays generated from a Cesium-137 source that penetrated these absorbers at various lengths, and then fitting the data to an exponential decay model to calculate the attenuation coefficients. In doing so, at this energy the attenuation coefficients for lead, copper, and aluminum were determined to be $0.93 \pm 0.19\text{cm}^{-1}$, $0.48 \pm 0.06\text{cm}^{-1}$, and $0.15 \pm 0.06\text{cm}^{-1}$ respectively. From this, values for the cross-sections of each material were obtained to be $2.82 \times 10^{-29} \pm 2.40 \times 10^{-30}$, $5.65 \times 10^{-30} \pm 2.33 \times 10^{-31}$, and $2.49 \times 10^{-30} \pm 2.67 \times 10^{-31}$ cm^2, respectively.

Introduction and Relevant Theory

The attenuation, or absorption, coefficient for a material at a given energy measures how easily a material can be penetrated by energetic/radioactive particles at said given energy. A large attenuation coefficient corresponds to the penetrating matter being quickly absorbed by the material, whereas a smaller attenuation coefficient corresponds to that material being relatively transparent to the penetrating entity. This has relevant applications to shielding and housing/transport of radioactive sources.

The relevant theory, the Beer-Lambert law[1], posits that the amount of particles that exist at a given finite length within an absorber decays exponentially. Formally, the attenuation coefficient μ corresponds to the decay factor in the following relationship:

$$N = N_0 e^{-\mu x} \tag{1}$$

where N_0 represents the number of penetrating particles present if the absorber were removed, x represents the length of a finite absorber that the beam must travel through, N represents the amount of penetrating particles present at that length, and e is Euler's Number.

The attenuation coefficient is also defined as

$$\mu = n\sigma \tag{2}$$

where n represents the atomic *number* density of the material and σ represents the cross-section of the material. Thus, a value of μ can be used along with information about a given material to calculate the cross-section via equation (2).

Experimental Setup and Results

As mentioned, the attenuation coefficients were calculated by measuring the net counts of gamma rays at 700KeV after the gamma rays had passed through an absorber of varying thicknesses. As shown in figure 1, the setup involved a Cesium-137 source placed 6 cm away from a gamma ray detector that would count the incoming gamma rays emitted from the source. Computer software was then used to isolate the gamma rays within the 650-700KeV range and tabulate those counts. Gamma rays were counted in ten second intervals, ten times each for each absorber length. A baseline measurement with no absorber (i.e. an absorber of length 0 cm) was counted as well, for each material. The length of the absorber was then incremented using segments of different sizes depending on the material available, with maximum absorber lengths within the 4-6.5 cm range.

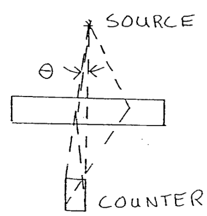

Figure 1: The experimental setup[1]

The following tables display the mean collected data of counts over ten trials vs. absorber length for copper, aluminum, and lead. Uncertainties for each measurement were calculated as $1/\sqrt{n}$ in accordance with Poisson statistics.

Table 1: Net counts vs. copper absorber length

absorber length (cm)	mean net counts	uncertainty
0	604.2	24.6
1.3	275.6	16.6
2.5	175.4	13.2
3.8	86.7	9.31
5.1	51.2	7.16

Table 2: Net counts vs. aluminum absorber length

absorber length (cm)	mean net counts	uncertainty
0	505.4	22.5
1.2	359.0	19.0
2.5	284.8	16.9
3.8	242.3	15.6
5.1	215.2	14.7
6.3	186.3	13.6

Table 3: Net counts vs. lead absorber length

absorber length (cm)	mean net counts	uncertainty
0	858.2	29.3
0.8	274.4	16.6
1.9	94.9	9.74
3.0	36.0	6.00
3.9	22.3	4.72

Analyzed Data

Taking the natural log of equation (1), we see that

$$\log(N) = \log(N_0) - \mu x \tag{3}$$

which is an equation linear in x. Thus, after the measurements, the natural log of the amount of counts was plotted vs. x, and then the data was fit to a linear fit in order to determine the parameters μ and N_0. An example of such a plot is shown in figure 2. In doing such analysis, the attenuation coefficients were determined to be $0.93 \pm 0.19cm^{-1}$, $0.48 \pm 0.06cm^{-1}$, and $0.15 \pm 0.06cm^{-1}$ for lead, copper, and aluminum respectively.

The accepted value of the attenuation coefficient for lead at 700KeV is 1.0014cm-1. Assuming a normal distribution of measured attenuation coefficients, and assuming the accepted value as the mean and the uncertainty as the standard deviation, the measured value is 0.38 standard deviations away from the mean. The accepted value of the attenuation coefficient for copper at 700KeV is 0.592cm-1. Performing the same analysis as above, the measured value is 1.87 standard deviations away from the mean. For aluminum, the accepted value is 0.185cm-1, which is 0.58 standard deviations away from the mean [2].

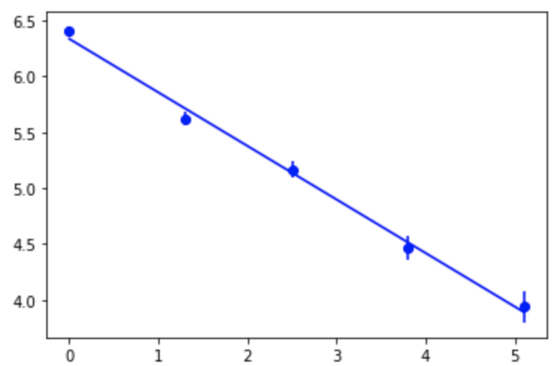

Figure 2: A plot of the log-fitted data for copper vs. length of absorber, with a best-fit line

Using equation 2 and these quantities, we can determine that the cross-sections for each material by dividing by the known number density for each element. For lead, copper, and aluminum the computed cross sections are $2.82\times 10^{-29}\pm 2.40\times 10^{-30}$, $5.65\times 10^{-30}\pm 2.33\times 10^{-31}$, and $2.49\times 10^{-30}\pm 2.67\times 10^{-31}$ cm^2.

We can then calculate how well the relevant theory holds by performing a chi-squared goodness of fit test on the fitted data, to see if it fits the model posited by the Beer-Lambert law. Performing a chi-squared goodness of fit test on the log-fitted data with a p-value of 0.05 for copper with the appropriate degrees of freedom, the chi-squared value is 5.65 for copper, 10.78 for aluminum, and 63.3 for lead.

Conclusions

The measured value of the attenuation coefficient for copper is quite far from the accepted value in the normal distribution framework, given that it is almost 2 standard deviations away from the mean. The accepted value and the measured value are most likely distinguishable and the accepted value was most likely not replicated in the experiment. On the other hand, the attenuation coefficients for aluminum and lead were much closer to the accepted values, albeit not exceptionally close. The measured values for each are at best indistinguishable from the accepted values and at worst distinguishable. In terms of testing the validity of the relevant theory, the Beer-Lambert law seems to hold for copper and aluminum in the experiment. The chi-squared values for these materials are lower than the critical values, suggesting a good fit-to-theory. However, for lead it appears that the relevant theory is inapplicable in the measured lab setting.

As far as limits on precision, the relevant statistical uncertainty is solely due to the number of counts in the Poisson regime, meaning that the only way to reduce relative uncertainty would be to measure more counts at each absorber length. Perhaps there were some calibration errors with the equipment. While it may be sobering, there is no real other conclusion than that the expected attenuation coefficient for copper was irreplicable in lab, and the Beer-Lambert law was at best inapplicable for lead in the lab.

Appendix

Consistent with the Poisson regime, for a given measured number of counts n the uncertainty in said measurement is given by $\sigma = 1/\sqrt{n}$. In propagating the uncertainty of the log-fitted counts, we propagate the uncertainty using the formula below:

$$\log(x \pm \sigma_x) = \log x \pm \frac{\sigma_x}{x} \tag{4}$$

The attenuation coefficients were calculated using the least squares polynomial fit method in the numpy python package.

References

[1] Introductory notes, Gamma Rays, Cornell Physics3310 Canvas website, accessed September 22, 2021.

[2] Databse of Attenuation Coefficients for Elements, National Institute of Standards and Technology Website, accessed September 22, 2021. https://www.nist.gov/pml/x-ray-mass-attenuation-coefficients